International Symposium on "Strangeness in Quark Matter 1997", April 14–18, Thera (Santorini), Hellas

Are strange stars distinguishable from neutron stars by their cooling behaviour?

Ch Schaab †, B Hermann, F Weber and M K Weigel

Institut für theoretische Physik, Ludwig-Maximilians Universität München, Theresienstr. 37, D-80333 München, Germany

Abstract. The general statement that strange stars cool more rapidly than neutron stars is investigated in greater detail. It is found that the direct Urca process could be forbidden not only in neutron stars but also in strange stars. If so, strange stars would be slowly cooling and their surface temperatures would be more or less indistinguishable from those of slowly cooling neutron stars. The case of enhanced cooling is reinvestigated as well. It is found that strange stars cool significantly more rapidly than neutron stars within the first ~ 30 years after birth. This feature could become particularly interesting if continued observation of SN 1987A would reveal the temperature of the possibly existing pulsar at its centre.

PACS numbers: 97.10.Cv, 97.60.Jd, 26.60+c, 12.38.Mh

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February 1, 2008

† E-mail address: schaab@gsm.sue.physik.uni-muenchen.de

1. Introduction

The theoretical possibility that strange quark matter – made up of roughly equal numbers of up, down and strange quarks – may be more stable than atomic nuclei (specifically iron, which is the most stable atomic nucleus) constitutes one of the most startling predictions of modern physics [1, 2, 3, 4, 5], which, if true, would have implications of greatest importance for laboratory physics, cosmology, the early universe, its evolution to the present day, and massive astrophysical objects [6]. Unfortunately it seems unlikely that lattice QCD calculations will be accurate enough in the foreseeable future to give a definitive prediction on the absolute stability of strange matter, so that one is presently left with experiments and astrophysical studies [7, 8, 9] to either confirm or reject the absolute stability of strange matter. In a recent investigation [10], dealing with the second item, we compared the cooling behaviour of neutron stars with the one of their hypothetical strange counterparts – strange stars [2, 11, 12, 13]. The theoretical predictions were compared with the body of observed data taken by ROSAT and ASCA. There have been investigations on this topic prior to this one (e.g., see [14, 15, 16, 17]). These, however, did not incorporate the so-called standard cooling scenario that turns out to be possible not only in neutron star matter but in strange quark matter too, altering some of the conclusions made in the earlier investigations significantly.

In the following section we will shortly review the structure of strange stars in comparison to neutron stars. Since the composition of strange matter turns out to be of great importance for the neutrino emissivity, we shall consider its determination in greater detail in section 3. The various neutrino emission processes and observational data are discussed in sections 4 and 5, respectively. In the last section, we present and discuss the results of cooling simulations and compare them with observed data.

2. Structure of strange stars and neutron stars

The cross section of a neutron star can be devided roughly into four distinct regimes. The outermost layer, with an optical depth of $\tau \sim 1$, is called photosphere. The thermal radiation, which can be observed by X-ray telescopes, is emitted from this region. This radiation dominates the cooling of pulsars older than $\sim 10^6$ yrs. The second regime is the star's outer crust, which consists of a lattice of atomic nuclei and a Fermi liquid of relativistic, degenerate electrons. The outer crust envelopes what is called the inner crust, which extends from neutron drip density, $\rho = 4.3 \times 10^{11}\,\mathrm{g\,cm^{-3}}$, to a transition density of about $\rho_{\rm tr} = 1.7 \times 10^{14}\,\mathrm{g\,cm^{-3}}$ [18]. Neutrons, both inside and outside of the atomic nuclei, are believed to form superfluid cooper pairs in this regime.

Beyond $\rho_{\rm tr}$ one enters the star's fourth regime, that is, its core where all atomic nuclei have dissolved into their constituents, protons, neutrons, and – due to the high Fermi pressure – possibly hyperons, more massive baryon resonances, and up, down and strange quarks. The latter possibility leads to so-called hybrid stars [19], which are to be distinguished from strange stars made up of absolutely stable atrange quark matter. Finally meson condensates may be found in the core, too. Neutrons and protons may form superfluid states in the core of a neutron star. It is however questionable whether or not the superfluids reach to the centre of the star. Solutions of the gap equation differ in the gap energy by almost one order of magnitude (see, for instance, [20, 21]). To account for these uncertainties, we shall investigate models

with and without superfluid cores.

In the first $\sim 10^6$ yrs the cooling of a neutron star is dominated by emission of neutrinos from the core. Depending on the possible neutrino reactions one can distinguish between standard and enhanced cooling (see section 4). Both the inner and outer crust act as a thermal insulator between the cooling core and the surface.

Since absolutely stable strange quark matter is selfbound, gravity is not necessary to bind strange stars, in contrast to neutron stars. As pointed out by Alcock, Farhi, and Olinto [12], a strange star can carry a solid nuclear crust whose density at its base is strictly limited by neutron drip. This is made possible by the displacement of electrons at the surface of strange matter, which leads to a strong electric dipole layer there. It is sufficiently strong to stabilize a gap between ordinary atomic (crust) matter and the quark-matter surface, which prevents a conversion of ordinary atomic matter into the assumed lower-lying ground state of strange matter. Obviously, free neutrons, being electrically charge neutral, cannot exist in the crust, because they do not feel the Coulomb barrier and thus would gravitate toward the strange-quark matter core, where they are converted by hypothesis into strange matter. Consequently, the density at the base of the crust (inner crust density) will always be smaller than neutron drip density. The main differences with respect to the structure of a neutron star is the composition of the core and the absence of the inner crust in the case of strange stars. As we will see the latter results in a smaller thermal insulation between the core and the surface of a strange star than in the case of a neutron star [14, 15].

3. Description of strange matter

We use the MIT bag model including $O(\alpha_\mathrm{s})$-corrections [22, 23] to model the properties of absolutely stable strange matter. Its equation of state and quark-lepton composition, which is governed by the conditions of chemical equilibrium and electric charge neutrality, is derived for that range of model parameters – that is, bag constant $B^{1/4}$, the strange quark mass m_s, and strong coupling constant α_s – for which strange matter is absolutely stable (i.e. energy per baryon E/A less than the one of ^{56}Fe, $E/A = 930$ MeV) (see also [24] where a different result for the composition was obtained).

In the limiting case of vanishing quark masses, electrons are not necessary to achieve charge neutrality. In the more realistic case of a finite strange quark mass m_s, the electrons can nevertheless vanish above a certain density, which depends on α_s. Figure 1 shows the allowed parameter space of α_s and $B^{1/4}$ for a fixed strange quark mass of $m_\mathrm{s} = 100$ MeV. This space is limited by two constraints. Firstly, the energy per baryon of three flavour quark matter has to be less than the one of iron (930 MeV), secondly, the energy per baryon of two flavour (up- and down-) quark matter has to be above the one of nucleons (938 MeV) minus a surface energy correction (~ 4 MeV) [23]. The almost horizontal lines represent the parameter sets for which the chemical potential of electrons and positrons are equal to their rest masses. In the region between these two lines electrons and positrons disappear even in the case of a nonvanishing strange quark mass. The behaviour of the electron's chemical potential depends on the chosen renormalization point ρ. We followed Duncan et al. [25] by renormalizing on shell ($\rho = m_\mathrm{s}$). The renormalization $\rho = 300$ MeV ($\approx \mu_\mathrm{s}$) suggested by Farhi and Jaffe [23] reduces the strangeness fraction and thus enhances the electron's chemical potential. The $Y_\mathrm{e} = 0$-region is therefore shifted to higher α_s-values. Since the MIT bag model is only phenomenlogical, it seems presumptous

to draw definitive conclusions for both cases.

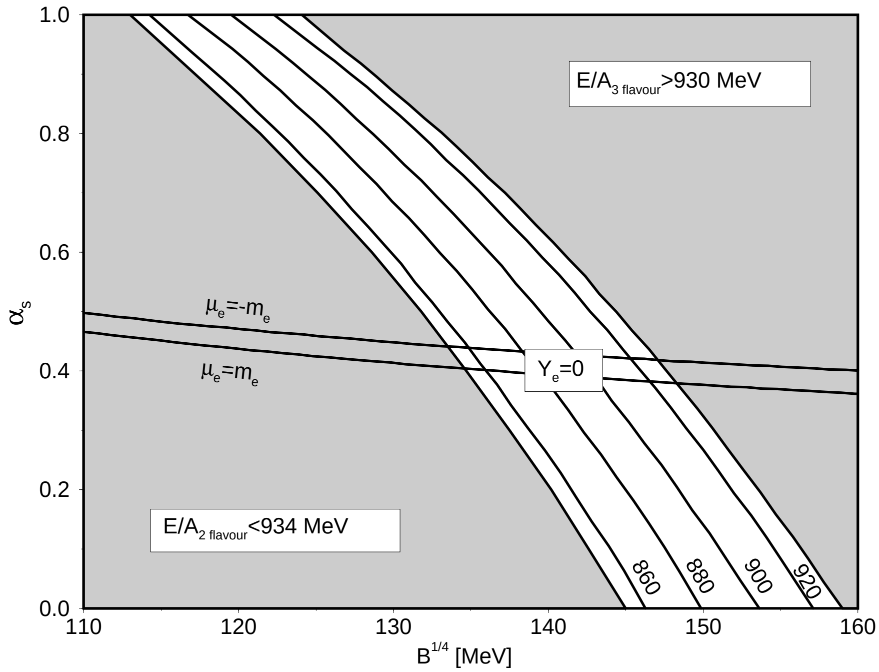

Figure 1. Allowed parameter range for which strange quark matter is absolutely stable. The shaded regions represent regions where either three flavour quark matter is not absolutely stable or nucleons would decay into quark matter. The labels attached to the four diagonal curves give the energy per baryon (in MeV) of three flavour quark matter. The two almost horizontal lines seperate the region where no electrons exist.

It was pointed out by Duncan et al. [25] (see also [12, 26]) that the neutrino emissivity of strange matter depends strongly on its electron fraction, $Y_{\rm e}$. For that reason we introduce two different, complementary parameter sets denoted SM-1 and SM-2, which correspond to strange matter that contains a relatively high electron fraction (SM-1, below the bottom line in figure 1), and $Y_{\rm e} = 0$ (SM-2, between the two lines) for the density range of interest here.

4. Neutrino emissivity

The neutrino emission processes can be divided into slow and fast ones (see table 1 for the most important reactions in the cores of neutron and strange stars). The large difference in the emissivities is caused by the rather different phase spaces associated with these reactions. The available phase space of the slow reactions is that of a two-baryon scattering process, whereas it is that of a one-baryon decay process for the fast reactions. The only fast processes in quark matter (the quark *direct* Urca processes)

$$\mathrm{d} \rightarrow \mathrm{u} + \mathrm{e}^- + \bar{\nu}_{\rm e} \tag{1}$$

and

$$\mathrm{s} \rightarrow \mathrm{u} + \mathrm{e}^- + \bar{\nu}_{\rm e}, \tag{2}$$

as well as their inverse reactions are only possible if the fermi momenta of quarks and electrons (p_F^i, $i = \mathrm{u, d, s; e^-}$) fulfill the so-called triangle inequality (e.g., $p_F^d < p_F^u + p_F^e$ for process (1)). This relation is the analogue to the triangle inequality established for nucleons and electrons in the nuclear matter case (nucleon direct Urca process [27, 28]).

Table 1. The most important neutrino emission processes in the cores of neutron stars and strange stars. The associated emissivities and constraints for the respective process are also given.

Process	Emissivity	Rapidity	Constraints
	Neutron stars		
modified Urca, e.g. $\mathrm{n + p + e^- \to n + n + \nu_e}$	$\sim 10^{21} \times T_9^8$	slow	
direct Urca, e.g. $\mathrm{p + e^- \to n + \nu_e}$	$\sim 10^{28} \times T_9^6$	fast	$Y_\mathrm{p} > 0.11$
K-, π-condensation	$\sim 10^{24-25} \times T_9^6$	fast	$n > n_\mathrm{c} \sim 3 - 5n_0$
	Strange stars		
modified Urca, e.g. $\mathrm{d + u + e^- \to d + d + \nu_e}$	$\sim 10^{20} \times T_9^8$	slow	$Y_\mathrm{e} > 0$
direct Urca, e.g. $\mathrm{u + e^- \to d + \nu_e}$	$\sim 10^{24} \times T_9^6$	fast	$Y_\mathrm{e} > 0$
Bremsstrahlung $q_1 + q_2 \to q_1 + q_2 + \nu + \bar{\nu}$	$\sim 10^{19} \times T_9^8$	slow	

If the electron fermi momentum is too small (i.e., Y_e is too little), then the triangle inequality for the processes (1) and (2) cannot be fulfilled and a bystander quark is needed to ensure energy and momentum conservation in the scattering process. The latter process is known as the quark *modified* Urca process, whose emissivity is considerably smaller than the emissivity of the direct Urca process. If the electron fraction vanishes entirely, as is the case for SM-2, both the quark direct and the quark modified Urca processes become unimportant. The neutrino emission is then dominated by bremsstrahlung processes only,

$$Q_1 + Q_2 \longrightarrow Q_1 + Q_2 + \nu + \bar{\nu}, \tag{3}$$

where Q_1, Q_2 denote any pair of quark flavours. For the emissivities associated with the quark direct Urca, quark modified Urca, and quark bremsstrahlung processes, we refer to references [25, 29, 30].

It has been suggested [31, 32] that the quarks eventually may form Cooper pairs. This would suppress, as in the nuclear matter case, the neutrino emissivities by an exponential factor of $\exp(-\Delta/k_\mathrm{B}T)$, where Δ is the gap energy, k_B Boltzmann's constant, and T the temperature. Unfortunately, up to now there exists neither a precise experimentally nor theoretically determined value for the gap energy. In order to give a feeling for the influence of a possibly superfluid behaviour of the quarks in strange matter, we choose $\Delta = 0.4$ MeV, as estimated in the work of Bailin and Love [31]. (Such a Δ value is not too different from the nuclear-matter case, where the proton $^1\mathrm{S}_0$ gap, for instance, amounts $\sim 0.2{-}1.0$ MeV [21, 33], depending on the nucleon-nucleon interaction and the microscopic many-body model.) The outcome of our superfluid strange matter calculations will be labeled SM-1^sf and SM-2^sf.

5. Observed data

Among the soft X-ray observations of the 23 sources which were identified as pulsars, the ROSAT and ASCA observations of PSRs 0002+62, 0833-45 (Vela), 0656+14,

0630+18 (Geminga) and 1055-52 (see table 2) achieved a sufficiently high photon flux such that the effective surface temperatures of these pulsars could be extracted by two- or three-component spectral fits [34]. The obtained effective surface temperatures, shown in figures 2 and 3, depend crucially on whether a hydrogen atmosphere is used or not. Since the photon flux measured solely in the X-ray energy band does not allow one to determine what kind of atmosphere one should use, we consider both the blackbody model and the hydrogen-atmosphere model, drawn in in Figs. 2 and 3 as error bars with a solid and hollow circle. The kind of atmosphere of individual pulsars could be determined by considering multiwavelength observations [35]. All error bars represent the 3σ error range due to the small photon fluxes.

Table 2. Surface temperatures as measured by an observer at infinity, T_s^∞, and spin-down ages, τ, of observed pulsars.

Pulsar	$\log\tau$ [yrs]	Model atmosphere	$\log T_s^\infty$ [K]	Reference
0002+62	$\sim 4\dagger$	blackbody	$6.20^{+0.09}_{-0.40}$	[36]
0833-45	$4.4 \pm 0.1\dagger$	blackbody	6.24 ± 0.08	[34]
(Vela)		magnetic H-atmosphere	5.88 ± 0.06	[37]
0656+14	5.05	blackbody	$5.89^{+0.08}_{-0.33}$	[38]
		magnetic H-atmosphere	$5.72^{+0.06}_{-0.03}$	[39]
0630+18	5.53	blackbody	$5.75^{+0.05}_{-0.08}$	[40]
(Geminga)		H-atmosphere	$5.42^{+0.12}_{-0.04}$	[41]
1055-52	5.73	blackbody	$5.90^{+0.09}_{-0.21}$	[38]

† estimated true age instead of spin-down age (see text).

Except for PSRs 0833-45 (Vela) and 0002+62, all ages are estimated by their spin-down age $\tau = P/2\dot{P}$. This relation implies however that both the moment of inertia and the magnetic surface field are constant with time, and that the braking index n is equal to its canonical value 3 (angular momentum loss dure to pure magnetic dipole radiation). The true ages may therefore be quite different from the spin-down ages. The age of Vela was recently determined in reference [42], and the approximate age of PSR 0002+62 is given by an estimate of the age of the related supernova remnant G 117.7+06.

6. Results and Discussion

The thermal evolution of strange stars and neutron stars was simulated using the evolutionary numerical code described in Schaab et al. [17] (see also [43, 44, 45, 46, 47]). The neutron star models are based on a broad collection of EOSs which comprises relativistic, fieldtheoretical equations of state as well as non-relativistic, Schroedinger-based ones (see [17] for details). As a specific feature of the relativistic models, they account for all baryon states that become populated in dense neutron star matter up to the highest densities reached in the cores of the heaviest neutron stars constructed from this collection of equations of state. Neutron stars are known to loose energy either via standard cooling or enhanced cooling. Both may be delayed by a superfluid core. Consequently all four options are taken into account here. These are labeled in figures 2 and 3 as NS-1 (enhanced cooling) and NS-2 (standard cooling) for normal

neutron star matter, and NS-1$^{\rm sf}$ and NS-2$^{\rm sf}$ (delayed cooling) for superfluid neutron star matter. The parameters of NS-1$^{\rm sf}$ and NS-2$^{\rm sf}$ are listed in table 4 of reference [17]. In analogy to this, the corresponding strange-star cooling curves are SM-1 (enhanced cooling) and SM-2 (standard cooling) for normal strange quark matter, and SM-1$^{\rm sf}$ and SM-2$^{\rm sf}$ (delayed cooling) for superfluid quark matter.

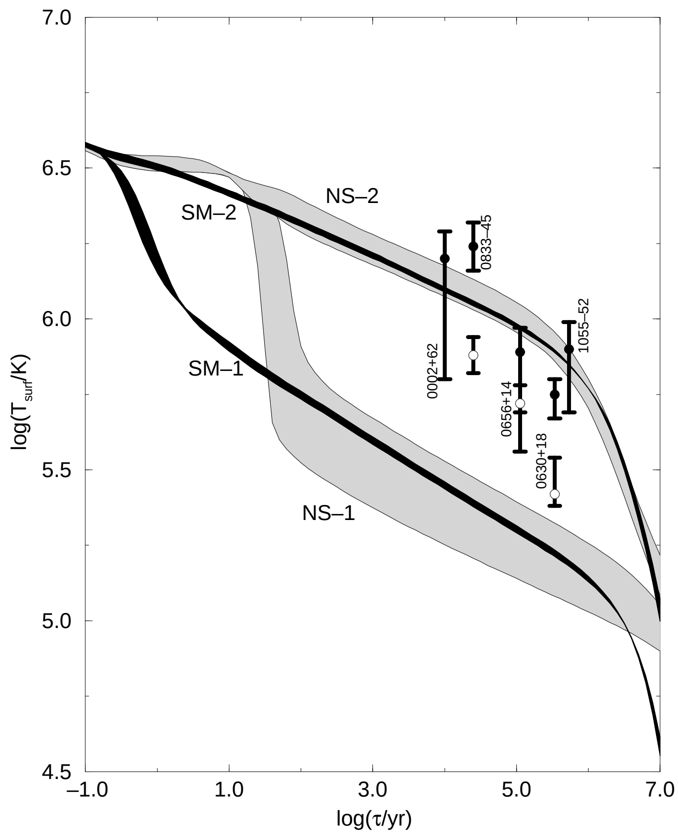

Figure 2. Cooling of non-superfluid strange star models SM-1 (lower solid band) and SM-2 (upper solid band), and neutron star models NS-1 (lower dotted band) and NS-2 (upper dotted band). The surface temperatures obtained with a blackbody- (magnetic hydrogen-) atmosphere are marked with error bars with solid (hollow) circle representing the most probable values (see table 2).

All calculations are performed for a gravitational star mass of $M = 1.4M_\odot$, about which the observed pulsar masses tend to scatter. The band-like structure of the cooling curves is supposed to reflect the uncertainties inherent in the equation of state of neutron-star and strange-star matter. These have their origin, in the case of neutron stars (gray bands), in the different many-body techniques used to solve the nuclear many-body problem. In the latter case, strange-star matter, the solid bands refer to the range of allowed bag values, $B^{1/4}$ (see figure 1), for which strange quark matter is absolutely stable. One might suspect that the large gap between the cooling tracks of SM-1 and SM-2 in figure 2 can be bridged steadily by varying the strong coupling constant $\alpha_{\rm s}$. However it turns out that this gap can be filled only for $\alpha_{\rm s}$-values that lie within an extremely small range. This is caused by the sensitive functional

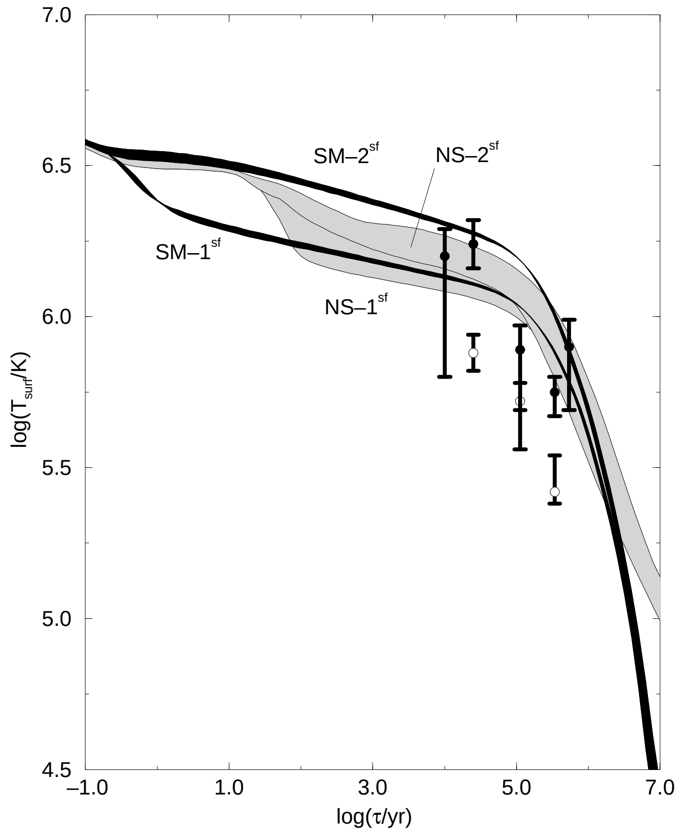

Figure 3. Cooling of superfluid strange star SM-1$^{\rm sf}$ (lower solid band) and SM-2$^{\rm sf}$ (upper solid band), and neutron star models NS-1$^{\rm sf}$ (lower dotted band) and NS-2$^{\rm sf}$ (upper dotted band).

relationship between $\alpha_{\rm s}$ and the neutrino luminosity L_ν, which is rather steep around that $\alpha_{\rm s}$-value for which the electrons vanish from the quark core of the star. All other values of $\alpha_{\rm s}$ give cooling tracks which are close to the upper or lower bands. This behaviour resembles the case of neutron stars, where the neutrino luminosity depends sensitively on the star's mass.

One sees from Figs. 2 and 3 that, except for the first ~ 30 years of the lifetime of a newly born pulsar, both neutron stars and strange stars may show more or less the *same* cooling behaviour, provided both types of stars are made up of either normal matter or superfluid matter. (We will come back to this issue below.) This is made possible by the fact that both standard cooling (NS-2) as well as enhanced cooling (NS-1) in neutron stars has its counterpart in strange stars too (SM-2 and SM-1, respectively). The point of time at which the surface temperature drop of a strange star occurs depends on the thickness of the nuclear crust that may envelope the strange matter core and thermaly insulate it from the surface [17]. In the present calculation, strange stars possess the densest possible nuclear crust, which is about 0.2 km thick. Thinner crusts would lead to temperature drops at even earlier times. Figures 2 and 3 indicate that the cooling data of observed pulsars do not allow to decide about the true nature of the underlying collapsed star, that is, as to whether it

is a strange star or a conventional neutron star. This could abruptly change with the observation of a very young pulsar shortly after its formation in a supernova explosion. In this case a prompt drop of the pulsar's temperature, say within the first 30 years after its formation, could offer a good signature of a strange star [14, 15]. This feature, provided it withstands a rigorous future analysis of the microscopic properties of quark matter, could become particularly interesting if continued observation of SN 1987A would reveal the temperature of the possibly existing pulsar at its centre.

Finally, we add some comments about the possibility that only the neutron star is made up of superfluid matter but not the strange star. In this case one has to compare the models SM-1 and SM-2 (see figure 2) with models NS-1^{sf} and NS-2^{sf} (see figure 3) yielding to an overall different cooling history of neutron stars and enhanced-cooling strange stars (SM-1). Therefore, the standard argument pointed out quite frequently in the literature that strange stars cool much more rapidly than neutron stars applies only to this special case.

Acknowledgments

We would like to thank J. Madsen for his comments.

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